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Thermoelectric device and optimum external load parameter and slenderness ratio

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ABSTRACT

Thermodynamic formulation of efficiency of thermoelectric power generator is presented in terms of the slenderness ratio $(X = (A_p/L_p)/(A_n/L_n))$ and the external load parameter $(Y = (R_L)/(L_n/k_{e,n}A_n))$. The optimum values of the slenderness ratio and external load parameter, maximizing the device efficiency, are formulated and predicted for several values of the thermal conductivity and the electrical conductivity ratios. The study is extended to include the influence of the dimensionless figure of merit (ZT_{avg}) and the temperature ratio on the maximum efficiency of the generator. It is found that for a fixed thermal conductivity ratio, the external load parameter increases with increasing the slenderness ratio while the electrical conductivity ratio of the p and n pins in the device reduces.

1. Introduction

Thermoelectric power generation can be considered one of the alternatives renewable energy resources. Since the scale of the energy transport is comparable to the atomic spacing within the thermoelectric material, energy transport generates non-equilibrium energy states between the lattice site and the electrons in the material. Moreover, energy absorbed from the external sources in the form of heat and electromagnetic waves results in electrons motion from hot region to cold region within the substrate material through which the Seebeck current is generated. Although the physical insight into the thermoelectric generation is involved with non-equilibrium energy transport, the efficiency and power generation of the device can be formulated in classical approaches. In this case, the problem simplifies and the optimum operation conditions maximizing the device efficiency can be obtained.

The theoretical energy conversion efficiency of a thermoelectric device depends on the thermoelectric figure of merit (Z), which is related to the material's Seebeck coefficient S, electrical conductivity σ, and thermal conductivity k via $Z = S^2\sigma/k$. Z has the unit of $1/K$ and it is usually combined with the average temperature of material (T), i.e. dimensionless figure of merit becomes ZT. In most of the applications, the figure of merit of thermoelectric materials has been limited to 1–1.5 and hence, thermoelectric power generators made from such materials have a low efficiency. On the other hand, thermal management of thermoelectric devices is critical for efficient operation. However, the performance of such thermoelectric generators can be improved significantly through using nanostructures [1]. The quantum and scattering effects in thin thermoelectric films result in reduction of the electrical conductivity leading to a significant low power factor of the cooling device [2]. The energy conversion efficiency in the heterojunction structures becomes an interest because of the heterostructures have an equivalent figure of merit higher than that corresponding to the homogeneous bulk material due to the non-equilibrium energy transport [3]. In addition, the non-equilibrium thermoelectric converters has a high thermoelectric power generation due to the high cooling efficiency, which can be possible through amplifying electron temperature across the interface in a forward structural configuration [4]. The interfacial effects on thermal and electrical transport in micro-thermoelectric coolers are the key issues, since the electron and phonon boundary resistance at the thermoelectric/metal interface as well as the non-equilibrium energy transport between electrons and phonons adjacent to the interface reduce the device cooling performance and decrease the Seebeck coefficient of the thermoelectric elements [5]. The improvement in the power density and efficiency of the radioisotropic thermoelectric generators is shown to be possible in the small thermoelectric devices [6].

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The investigation of the effect of thermoelectric properties on the energy conversion efficiency of the thermoelectric device is necessary using the new thermal rate equations [7]. The solar receiver can be used to achieve a high conversion efficiency of such devices [8]. Although extensive research studies have been reported in the literature to examine the energy conversion efficiency of the thermoelectric power generators through improving thermoelectric properties of the device [1–8], the geometric configuration and the external load parameter of the device on the conversion efficiency has not been studied in details [9–11]. Therefore, in the present study, thermodynamic analysis of thermoelectric power generating device consisting of p and n semi-conductor pins is considered and the influence of the slenderness ratio and the external load parameter on the thermoelectric power and device efficiency is presented.

2. Efficiency analysis of thermoelectric power generator

The thermoelectric power generator consists of p and n semi-conductor pins, which operates between the hot and cold junctions. The schematic view of a typical thermoelectric power generator is shown in Fig. 1. The thermodynamic efficiency of the power generator can be written as:

$$\eta = \frac{I^2 R_L}{\alpha I T_1 + K(T_1 - T_2) - \frac{1}{2} I^2 R} \tag{1}$$

where

$$I = \frac{\alpha (T_1 - T_2)}{R_L + R}, \tag{2}$$

$$\alpha = \alpha_p - \alpha_n, \tag{3}$$

$$K = \frac{A_p k_p}{L_p} + \frac{A_n k_n}{L_n}, \tag{4}$$

and,

$$R = \frac{L_p}{A_p k_{e,p}} + \frac{L_n}{A_n k_{e,n}}. \tag{5}$$

Introducing the following dimensionless quantities:

$$X = \frac{A_p/L_p}{A_n/L_n} (\text{Slenderness ratio, geometric parameter}) \tag{6}$$

$$Y = \frac{R_L}{L_n/(k_{e,n} A_n)}, \ (\text{External load parameter}) \tag{7}$$

$$r_k = \frac{k_p}{k_n}, \ (\text{Thermal conductivity ratio}) \tag{8}$$

$$r_{ke} = \frac{k_{e,p}}{k_{e,n}}, \ (\text{Electrical conductivity ratio}) \tag{9}$$

$$\theta = \frac{T_2}{T_1}, \ (\text{Temperature ratio}) \tag{10}$$

And, (the Figure of merit)

$$ZT_{ave} = \frac{\alpha^2}{\left(\sqrt{\frac{k_n}{k_{e,n}}} + \sqrt{\frac{k_p}{k_{e,p}}}\right)^2} \left(\frac{T_1 + T_2}{2}\right) \tag{11}$$

The efficiency (eq. (1)) as function of the above six dimensionless parameters becomes

$$\eta = (1-\theta) \frac{2ZT_{ave}\left(1 + \sqrt{\frac{r_k}{r_{ke}}}\right)^2}{(1+\theta) Y (r_k X + 1)\left(1 + \frac{R}{R_L}\right)^2 + 2ZT_{ave}\left(1 + \sqrt{\frac{r_k}{r_{ke}}}\right)^2 \left[1 + \left(\frac{1+\theta}{2}\right)\left(\frac{R}{R_L}\right)\right]} \tag{12}$$

where

$$\frac{R}{R_L} = \frac{1}{Y}\left(\frac{1}{r_{ke} X} + 1\right). \tag{13}$$

3. Thermal optimization

For selected n-type and p-type thermoelectric materials and operation temperatures, the dimensionless parameters: r_k, r_{ke},

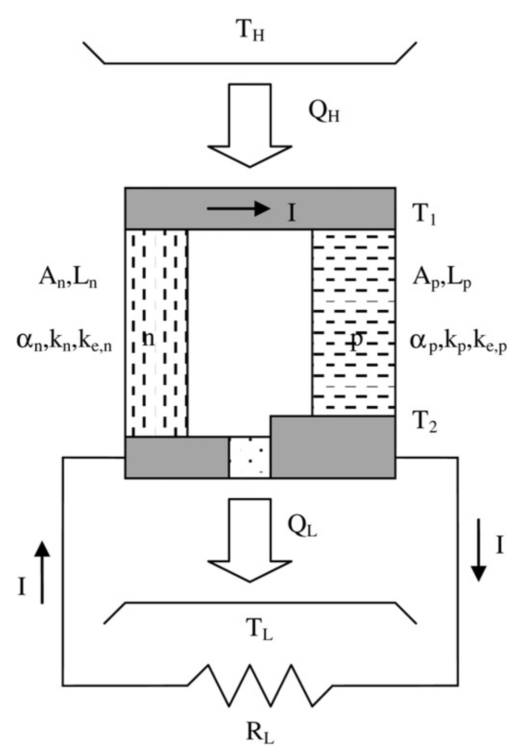

Fig. 1. Schematic view of thermoelectric power generator.

ZT_{ave}, and θ can be fixed. Then, the efficiency can be maximized with respect to the slenderness ration (X, geometric parameter) and the external load parameter (Y). In this case, the maximum efficiency with respect to these parameters can be obtained as:

$$\left\{\begin{array}{l}\frac{\partial\eta}{\partial X}=0\\ \frac{\partial\eta}{\partial Y}=0\end{array}\right\}\rightarrow\left\{\begin{array}{c}X_{opt}=\frac{1}{\sqrt{r_k r_{ke}}}\\ Y_{opt}=\sqrt{1+ZT_{avg}}\left(1+\sqrt{\frac{r_k}{r_{ke}}}\right)\end{array}\right\} \tag{14}$$

After substituting eq. (14) into eq. (12), the maximum efficiency of the thermoelectric generator becomes

$$\eta_{max}=(1-\theta)\frac{\sqrt{1+ZT_{avg}}-1}{\sqrt{1+ZT_{avg}}+\theta} \tag{15}$$

Since the Carnot efficiency is $\eta_{Carnot}=(1-\theta)$, then

$$\frac{\eta_{max}}{\eta_{Carnot}}=\frac{\sqrt{1+ZT_{avg}}-1}{\sqrt{1+ZT_{avg}}+\theta} \tag{16}$$

The special case can be obtained after setting the Carnot efficiency 1, i.e. $\theta=0$, the absolute maximum efficiency can be obtained.

A computer program is developed to predict the influence of the slenderness ratio and the external load parameter on the efficiency of the thermoelectric generator (eqs. (12) and (16)).

4. Results and discussion

Thermal analysis of thermoelectric power generator consisting of p and n-type semi-conductor pins is considered. The thermal efficiency and power generation are formulated. The influence of the slenderness ratio, associated with the geometric configuration of the thermoelectric generator, and the external load parameter on the maximum efficiency is examined.

Fig. 2 shows the variation of efficiency with the slenderness ratio (eq. (6)) for various external load parameters (eq. (7)). It should be noted that the slenderness ratio is associated with the ratio of area to height of the semi-conductors while the external load parameter is related to the external load connected to the generator. Increasing the slenderness ratio (X, where$(X=(A_p/L_p)/(A_n/L_n))$) towards 1 increases the efficiency irrespective of the values of the load ratio considered. This behavior is associated with eq. (12), in which case, the term $Y(r_kX+1)[1+1/Y(1/r_{k,e}X+1)]^2$ becomes small with increasing the slenderness ratio (X). Consequently, slenderness ratio in the range of $0\le X\le 1$ enhances the efficiency; in which case, the slenderness ratio of p-type semi-conductor (A_p/L_p) is less than the slenderness ratio of n-type semi-conductor. The rate of increase in the efficiency changes with the slenderness ratio in such a way that the rate of this increase enhances with increasing the load parameter. The maximum efficiency occurs at different slenderness ratios for different external load parameters. This is because of the non-linear behavior of the efficiency with the slenderness ratio and the external load parameter (eq. (15)). Consequently, a unique value of the maximum efficiency occurs for a particular combination of the slenderness ratio and the external load parameter. However, the efficiency reduces gradually with further increasing the slenderness ratio. This is attributed to a non-linear relation between the efficiency, the slenderness ratio, and the external load parameter (eq. (13)). This situation can also be observed from Fig. 3 in which 3-dimensional plot of the efficiency with the slenderness ratio and the load parameter is shown.

Fig. 4 shows the maximum efficiency with temperature ratio for the fixed slenderness ratio ($X=1$) and the external load parameter ($Y=3$). The maximum efficiency reduces with increasing the temperature ratio ($\theta=T_2/T_1$). The maximum efficiency is associated with the Carnot efficiency, which is $(1-\theta)$, (eq. (15)); therefore, increasing the temperature ratio lowers the Carnot efficiency and consequently the maximum efficiency. It should be noted that the maximum efficiency is always less than the Carnot efficiency; in which case $(\sqrt{1+ZT_{avg}}-1)<(\sqrt{1+ZT_{avg}}+\theta)$ (eq. (16)). Moreover, the decay rate of the maximum efficiency is not linear. Increasing the dimensionless the figure of merit, ZT_{avg}, enhances the maximum efficiency. This can be seen from Fig. 5, in which 3-dimensional plot of the maximum efficiency with the temperature ratio and ZT_{avg} is shown. For the practical applications point of view, the maximum ZT_{avg} is about 2, which in turn results the maximum efficiency in the order of 20% for the temperature ratio of 0.5. Reducing the temperature ratio further does not result in excessive increase of the maximum efficiency, e.g. the maximum efficiency is in the order of 0.35 for $ZT_{avg}=2$ and$\theta=0$. This clearly indicates that the maximum efficiency achievable is limited to 0.2–0.25 for $ZT_{avg}=2$. However, further reduction in ZT_{avg} lowers the maximum efficiency.

In order to assess the optimum values for the slenderness ratio and the load parameter, eq. (14) is derived. Fig. 6 shows the

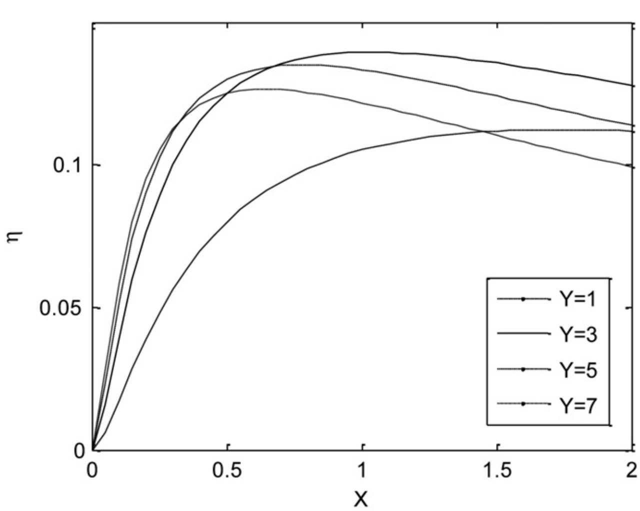

Fig. 2. Variation of efficiency with the slenderness ratio for different external load parameters. $\theta=0.5$, $r_k=1.0$, $r_{ke}=1.0$ and $ZT_{ave}=1.5$.

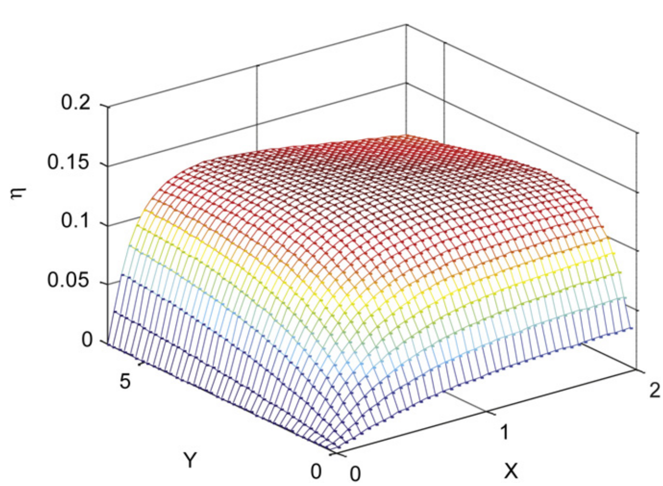

Fig. 3. 3-dimensional view of the variation of efficiency with the slenderness ratio and the external load parameter. $\theta=0.5$, $r_k=1.0$, $r_{ke}=1.0$ and $ZT_{ave}=1.5$.

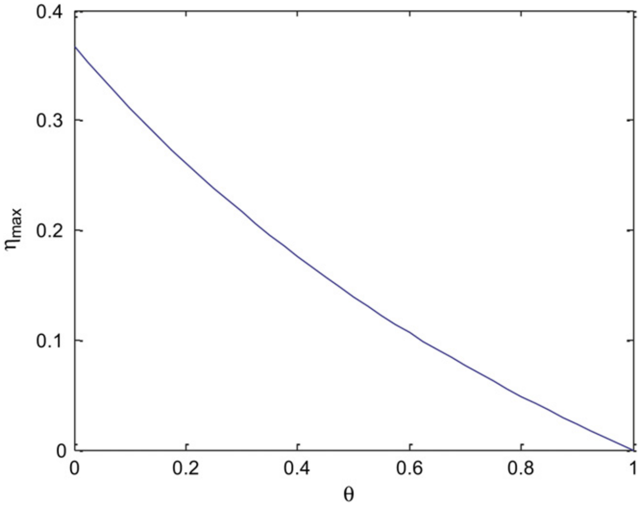

Fig. 4. Variation of the maximum efficiency with the temperature ratio.

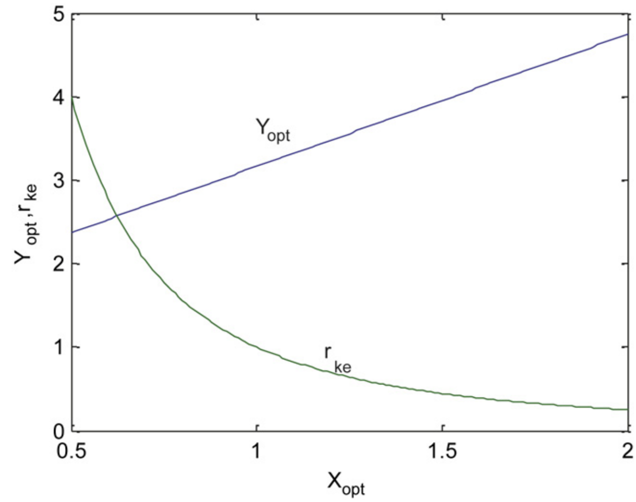

Fig. 6. The external load parameter versus the slenderness ratio for varying electrical conductivity ratio ($ZT_{ave} = 1.5$, $r_k = 1.0$).

variation of the optimum external load parameter with the optimum slenderness ratio for varying the electrical conductivity ratio while keeping the thermal conductivity ratio constant ($r_k = 1.0$). It should be noted that both the optimum external load parameter and slenderness ratio are the functions of the electrical conductivity ratio (eq. (14)). In addition, the optimum load parameter and the slenderness ratio are obtained through maximizing the device efficiency. It is evident that the optimum load parameter increases with increasing the optimum slenderness ratio. In this case, the electrical conductivity ratio is decreasing with increasing the slenderness ratio. Moreover, the variation of the optimum load parameter with the optimum slenderness ratio is linear while the variation of the electrical conductivity ratio with the optimum slenderness ratio is non-linear. Consequently, fixing the electrical conductivity ratio for *p* and *n* semi-conductor pins in the thermoelectric element there exist a unique optimum slenderness ratio for the optimum operating external load parameter. In addition, to keep the slenderness ratio optimum for the optimum load parameter, the electrical conductivity ratio of *p* and *n* semi-conductor pins in the thermoelectric device should be varied in a non-linear fashion.

Fig. 7 shows the optimum external load parameter with the optimum slenderness ratio for varying the thermal conductivity ratio while keeping the electrical conductivity ratio constant ($r_{ke} = 1.0$). The optimum external load parameter decreases with increasing the optimum slenderness ratio unlike that occurs in Fig. 6. In this case, the thermal conductivity ratio reduces with increasing the optimum slenderness ratio. It should be noted that the value of the optimum external load parameter vary around 2.5–5 for both Figs. 6 and 7. Consequently, the selection of the thermal conductivity ratio and the electrical conductivity ratio is important for setting the optimum external load parameter for the optimum slenderness ratio. The variation of the optimum external load parameter with the optimum slenderness ratio is in non-linear form. Therefore, when designing the thermoelectric power generator device for the specific external load parameter, the selection of the ratios of thermal conductivity and the electrical conductivity of *p* and *n* semi-conductor pins are very crucial. In addition, for the optimal operation of the device, there exist sets of slenderness ratio, which depend on the ratios of the thermal conductivity and the electrical conductivity of the semi-conductor pins in the device.

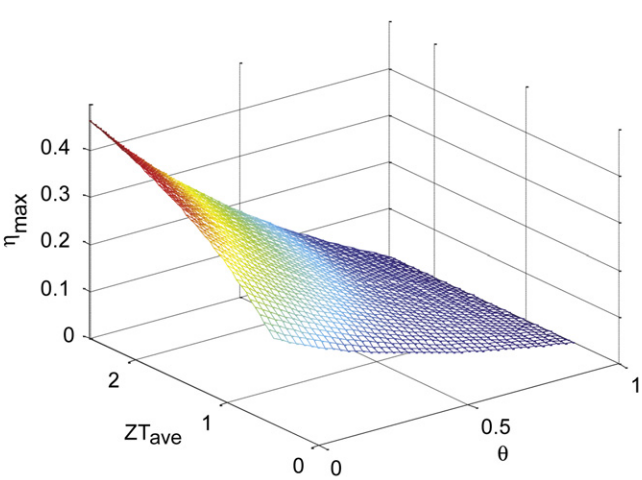

Fig. 5. 3-dimensional view of the maximum efficiency with ZT_{ave} and the temperature ratio $r_k = 1.0$ and $r_{ke} = 1.0$.

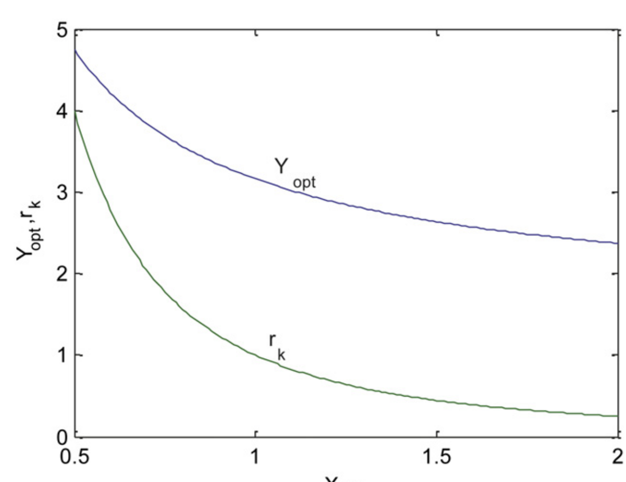

Fig. 7. The external load parameter versus the slenderness ratio for varying thermal conductivity ratio ($ZT_{ave} = 1.5$, $r_{ke} = 1.0$).

5. Conclusion

Efficiency analysis of the thermoelectric power generator is considered and the influence of the slenderness ratio and the external load parameter on the maximum efficiency is examined. In the analysis, the thermoelectric power generator consisting of a single cell of *p*- and *n*-types is considered. It is found that the efficiency attains high values for the slenderness ratio less than 1 for almost all the external load parameters considered, which is more pronounced for the large values of the external load parameter. This indicates that the thermal efficiency can be improved while lowering the slenderness ratio for large external load parameters, provided that the relation among the efficiency, the slenderness ratio, and the external load parameter is non-linear. For the practical applications, the maximum dimensionless figure of merit (ZT_{avg}) is in the order of 2 and the corresponding maximum efficiency is only about 20%. Although increasing the temperature ratio improves the maximum efficiency, this may not be very practical from the application point of view. The optimum external load parameter varies with the optimum slenderness ratio. However, this variation depends on the thermal and the electrical conductivity ratios of *p* and *n* semi-conductor pins in the thermoelectric power generator. In this case, keeping the thermal conductivity ratio constant while reducing the electrical conductivity ratio, the external load parameter increases linearly with increasing the optimum slenderness ratio. However, keeping the electrical conductivity ratio constant while reducing the thermal conductivity ratio, the optimum external load parameter reduces in a non-linear fashion with increasing the slenderness ratio.

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Nomenclature

A_n: cross sectional area of *n*-type semi-conductor (m^2)
A_p: cross sectional are of *p*-type semi-conductor (m^2)
I: electrical current (A)
$k_{e,n}$: electrical conductivity of *n*-type semi-conductor (S/m)
$k_{e,p}$: electrical conductivity of *p*-type semi-conductor (S/m)
k_n: thermal conductivity of *n*-type semi-conductor (W/mK)
k_p: thermal conductivity of *p*-type semi-conductor (W/mK)
K: overall thermal conductivity of the thermoelectric generator, $K = (A_p k_p / L_p) + (A_n k_n / L_n)$
L_n: length of *n*-type semi-conductor (m)
L_p: length of *p*-type semi-conductor (m)
r_k: thermal conductivity ratio
r_{ke}: electric conductivity ratio
R: overall electrical resistivity of the thermoelectric generator, $R = (L_p / A_p k_{e,p}) + L_n / A_n k_{e,n}$
R_L: external electrical load (Ω)
T_1: hot side temperature of the thermoelectric generator (K)
T_2: cold side temperature of the thermoelectric generator (K)
X: slenderness ratio $X = (A_p/L_p)/(A_n/L_n)$
Y: external load parameter $Y = R_L/L_n/(k_{e,n}A_n)$
Z: figure of merit, $Z = \alpha^2/KR$ (1/K)
α: total Seebeck coefficient, $\alpha = \alpha_p - \alpha_n$(V/K)
α_p: Seebeck coefficient of the *p*-type semi-conductor (V/K)
α_n: Seebeck coefficient of the *n*-type semi-conductor (V/K)
η: efficiency
η_{Carnot}: Carnot cycle efficiency
θ: dimensionless temperature $= T_2/T_1$